

Patterns and partners for chiral symmetry restoration

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We present and analyze a new set of Ward Identities which shed light on the distinction between different patterns of chiral symmetry restoration in QCD, namely $O(4)$ vs $O(4) \times U(1)_A$. The degeneracy of chiral partners for all scalar and pseudoscalar meson nonet members are studied through their corresponding correlators. In particular, we conclude that the anomalous $U(1)_A$ symmetry has to be fully restored at the critical temperature for two massless quark flavours. Our analysis supports ideal mixing between the η - η' and the $f_0(500)$ - $f_0(980)$ mesons at $O(4) \times U(1)_A$ restoration. On the massive case, if $O(4)$ and $U(1)_A$ transitions are separated, we find a temperature range where the pseudoscalar mixing vanishes. We test our results against lattice data and provide further relevant observables regarding chiral and $U(1)_A$ restoration for future lattice and model analyses.

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INTRODUCTION

Chiral symmetry restoration is a prominent feature of the QCD phase diagram, realized in lattice simulations and presumably in matter formed after a Heavy Ion Collision. For a vanishing baryon density, a crossover transition for physical masses and $N_f = 2+1$ flavors is expected at a transition temperature $T_c \sim 155$ MeV [1–3], signaled by the inflection of the light quark condensate $\langle \bar{q}q \rangle_l$ and the peak of the scalar susceptibility. For two massless flavors (chiral limit) a phase transition takes place with vanishing condensate and divergent susceptibility [4].

In addition, the anomalous axial $U(1)_A$ symmetry can be asymptotically restored, driven by the vanishing of the instanton density [5]. An ongoing debate is whether $U(1)_A$ is then restored at the chiral transition. If so, the restoration pattern would be $O(4) \times U(1)_A$ instead of $SU(2)_L \times SU(2)_R \sim O(4)$ for two massless flavors and the order of the transition would change from second to first order [6, 7]. The restoration of $U(1)_A$ also affects the chiral transition order for three flavors [8], as well as the behaviour near the critical end point at finite temperature and baryon density [9].

The implications for the hadron spectrum are crucial. The restoration of a global symmetry implies a degeneracy in the spectrum of particles, which is customarily studied through the behavior of their correlation functions, their associated susceptibilities or screening masses. These correlators are meant to be very sensitive to the transition from the ordered to the disordered state. The hadronic states becoming degenerate at chiral restoration are usually known as chiral partners. In more detail, the pion is expected to degenerate with the $\sigma/f_0(500)$ meson within a $O(4)$ pattern, whereas the restoration of the $U(1)_A$ symmetry would also degenerate the pion and the $a_0(980)$, i.e. the member of the

scalar nonet with the same pion quantum numbers but an opposite parity. In this context, it is also natural to investigate the fate of the rest of the members of the scalar and pseudoscalar nonet, i.e the $\kappa/K_0(800)$ versus the kaon for $I = 1/2$, and the $f_0(980) - f_0(500)$ pair versus the $\eta - \eta'$ for $I = 0$.

Moreover, if chiral and $U(1)_A$ restoration happen to be close, a proper description of light meson phenomenology at finite temperatures will require the inclusion of the η' as the ninth Goldstone boson [10]. In fact, there is experimental evidence of the reduction of the η' mass in the hot medium [11], pointing out to $U(1)_A$ restoration. This should result in an increase of the η' production cross section, which might be observed in dilepton and diphoton experiments at finite temperature [12].

Several effective models and renormalization-group approaches have been carried out to analyze these issues [7–9, 13–17]. In addition, chiral partners and patterns have been recently examined by different lattice collaborations. Nevertheless, there is currently no consensus on the restoration scenario. On the one hand, a $O(4)$ pattern has been proposed in [3]. Namely, $U(1)_A$ symmetry, studied through $\pi - a_0$ degeneration, is restored well above T_c , the point where $\pi - \sigma$ degeneration occurs. On the other hand, a $O(4) \times U(1)_A$ pattern for two flavors has been suggested in [18, 19] in the chiral limit and in [20] for the massive case.

Our aim in this work is to provide new and model-independent results to shed light on chiral patterns and partner degeneration. These results will be testable in lattice and in model analyses and will help to solve this current controversy. For that sake, we will rely on Ward Identities (WI) derived formally in QCD. Their consequences for chiral restoration of the full meson nonet, including the $I = 1/2$ sector, will be analyzed in the next sections.

WARD IDENTITIES

We will start considering an infinitesimal vector and axial transformation on a quark field $\psi' = \psi + \delta\psi$,

$$\delta\psi(x) = i \left(\alpha_V^a(x) \frac{\lambda_a}{2} + \alpha_A^a(x) \frac{\lambda_a}{2} \gamma_5 \right) \psi(x),$$

with $\lambda^{a=1,\dots,8}$ the $SU(3)$ Gell-Mann matrices and $\lambda^0 = \sqrt{2/3} \mathbb{1}$. The expectation value of a pseudoscalar operator \mathcal{O} in terms of the transformed fields leads to [21]

$$\left\langle \frac{\delta\mathcal{O}(y)}{\delta\alpha_A^a(x)} \right\rangle = - \left\langle \mathcal{O}(y) \bar{\psi}(x) \left\{ \frac{\lambda^a}{2}, \mathcal{M} \right\} \gamma_5 \psi(x) \right\rangle \quad (1)$$

$$+ i \frac{\delta_{a0}}{\sqrt{6}} \langle \mathcal{O}(y) A(x) \rangle,$$

$$\left\langle \frac{\delta\mathcal{O}(y)}{\delta\alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}(y) \bar{\psi}(x) \left[\frac{\lambda^a}{2}, \mathcal{M} \right] \psi(x) \right\rangle, \quad (2)$$

with $A(x) = \frac{3g^2}{16\pi^2} \text{Tr}_c G_{\mu\nu} \tilde{G}^{\mu\nu}$ the anomalous divergence of the $U(1)_A$ current and $\mathcal{M} = \text{diag}(\hat{m}, \hat{m}, m_s)$ the quark mass matrix. In the following, we denote as $\pi^a = i\bar{\psi}_l \gamma_5 \tau^a \psi$ and $\delta^a = \bar{\psi}_l \tau^a \psi_l$, with $a = 1, 2, 3$ and ψ_l the light quark doublet, the isotriplet pseudoscalar (pion) and scalar ($a_0(980)$) bilinears, with $P_{\pi\pi}$ and $S_{\delta\delta}$ their corresponding euclidean finite- T correlators. The $I = 1/2$ pseudoscalar and scalar bilinears are $K^a = i\bar{\psi}_l \gamma_5 \tau^a \psi$ and $\kappa^a = \bar{\psi}_l \tau^a \psi_l$ respectively, with $a = 4, \dots, 7$, and P_{KK} and $S_{\kappa\kappa}$ their correlators. Likewise, $\eta_l = i\bar{\psi}_l \gamma_5 \psi_l$, $\eta_s = i\bar{s} \gamma_5 s$, $\sigma_l = i\bar{\psi}_l \psi_l$ and $\sigma_s = i\bar{s} s$ denote the light- and strange-quark part of the isosinglet bilinears, with correlators X_{ll} , X_{ls} and X_{ss} , with $X = P, S$. Note that η_l and η_s mix to give the physical η and η' . Similarly, the mixing of σ_l and σ_s generates the $f_0(500)$ and $f_0(980)$ resonances.

Applying (1) to a single bilinear $\mathcal{O}^a = i\bar{\psi} \gamma_5 \lambda^a \psi$, one obtains a set of WI relating quark condensates and pseudoscalar susceptibilities $\chi_P^{ab}(T)$, already described in [21]. Note that $\chi_P^{ab}(T) = P_{ab}(p = 0; T)$ [22]. The difference between π and η_l WIs leads to

$$\chi_P^{ls}(T) = -2 \frac{\hat{m}}{m_s} \chi_{5, \text{disc}}(T) = \frac{1}{2\sqrt{3}} \frac{1}{\hat{m} - m_s} \chi_P^{8A}(T), \quad (3)$$

where $\chi_{5, \text{disc}} = \frac{1}{4} (\chi_P^\pi - \chi_P^{\eta_l})$ is customarily the parameter measuring $O(4) \times U(1)_A$ restoration [3]. Consistently, in (3) it is given as a pure anomalous contribution. This relation is testable in lattice and model analyses and, as we are about to see, plays an important role regarding chiral pattern restoration.

The evaluation of (2) becomes non trivial only in the $I = 1/2$ channel. Combining this result with the kaon WI [21] gives

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2}{m_s^2 - \hat{m}^2} [m_s \langle \bar{q}q \rangle_l(T) - 2\hat{m} \langle \bar{s}s \rangle(T)], \quad (4)$$

which establishes a relation for K - κ degeneration.

Considering in (1) a two-point function $\mathcal{O}^{ab} = P^a S^b$, with P and S generic pseudoscalar and scalar bilinears connected by $SU(2)_A$ transformations, we get additional WIs

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m} \int_T dx \langle \mathcal{T} \sigma_l(y) \pi(x) \pi(0) \rangle, \quad (5)$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m} \int_T dx \langle \mathcal{T} \delta(y) \pi(x) \eta_l(0) \rangle, \quad (6)$$

$$P_{ls}(y) = \frac{1}{3} \hat{m} \int_T dx \langle \mathcal{T} \eta_s(y) \pi(x) \delta(0) \rangle, \quad (7)$$

$$S_{ls}(y) = -\frac{1}{3} \hat{m} \int_T dx \langle \mathcal{T} \sigma_s(y) \pi(x) \pi(0) \rangle, \quad (8)$$

$$d^{abc} [P_{KK}(y) - S_{\kappa\kappa}(y)] = \hat{m} \int_T dx \langle \mathcal{T} K^b(y) \kappa^c(x) \pi^a(0) \rangle, \quad (9)$$

with $\int_T dx \equiv \int_0^{1/T} d\tau \int d^3\vec{x}$, d^{abc} the symmetric $SU(3)$ coefficients, $a = 1, 2, 3$, $b, c = 4, \dots, 7$. These equations parameterize the degeneration of $SU(2)_A$ chiral partners in terms of three-point functions; the latter encode the physical vertices responsible for the breaking of such a degeneracy. Furthermore, if P and S are linked through a $U(1)_A$ transformation, (1) gives rise to

$$P_{\pi\pi}(y) - S_{\delta\delta}(y) = \int_T dx \langle \mathcal{T} \pi(y) \delta(0) \tilde{\eta}(x) \rangle, \quad (10)$$

$$P_{ll}(y) - S_{ll}(y) = \int_T dx \langle \mathcal{T} \eta_l(y) \sigma_l(0) \tilde{\eta}(x) \rangle, \quad (11)$$

$$P_{ls}(y) - S_{ls}(y) = \int_T dx \langle \mathcal{T} \eta_l(y) \sigma_s(0) \tilde{\eta}(x) \rangle, \quad (12)$$

$$P_{ss}(y) - S_{ss}(y) = \int_T dx \langle \mathcal{T} \eta_s(y) \sigma_s(0) \tilde{\eta}(x) \rangle, \quad (13)$$

$$P_{KK}(y) - S_{\kappa\kappa}(y) = \int_T dx \langle \mathcal{T} K(y) \kappa(0) \tilde{\eta}(x) \rangle, \quad (14)$$

where $\tilde{\eta}(x) = \hat{m} \eta_l(x) + m_s \eta_s(x) + \frac{1}{2} A(x)$. These new equations include now explicit (m_s) and anomalous (A) terms responsible for $U(1)_A$ breaking.

CONSEQUENCES FOR CHIRAL SYMMETRY RESTORATION

$SU(2)_A$ transformations mix $\pi - \sigma_l$ and $\delta - \eta_l$ states. Hence, if chiral symmetry is exactly restored, one can rotate π to σ_l and η_l to δ , so that their correlators become degenerate, i.e. $P_{\pi\pi} \stackrel{O(4)}{\sim} S_{ll}$, $P_{ll} \stackrel{O(4)}{\sim} S_{\delta\delta}$. On the other hand, $U(1)_A$ rotations allow to connect bilinears with opposite parity, for instance $\pi - \delta$ and $\eta_l - \sigma_l$. In a fully $U(1)_A$ restored scenario, pseudoscalar and scalar correlators degenerate so that $P_{\pi\pi} \stackrel{U(1)_A}{\sim} S_{\delta\delta}$, $P_{ll} \stackrel{U(1)_A}{\sim} S_{\sigma\sigma}$.

These four correlators have been actively investigated in lattice and theoretical analyses to study partner degeneration [3, 16–20, 23, 24]. In a $O(4)$ pattern, $\pi - \sigma_l$ and $\delta - \eta_l$ would become chiral partners, while $\pi - \delta - \sigma_l - \eta_l$ would degenerate in the $O(4) \times U(1)_A$ one.

Let us start by analyzing partner degeneration through the study of the WIs (5)–(9) and (10)–(14). These equations impose constraints on specific model calculations, connecting two-point functions (correlators and susceptibilities) with three-point ones (vertices) and could be tested in lattice analyses. Moreover, their r.h.s. can be related to meson scattering processes. More specifically, the coupling of the $\sigma_{l,s}$ bilinears to an external scalar source in QCD is expressed in the meson lagrangian into the $\pi\pi$, $\bar{K}K$ and $\eta\eta$ channels [25]. Therefore, the r.h.s. of identities (5) and (8) are directly related to $\pi\pi \rightarrow \pi\pi$, $\bar{K}K \rightarrow \pi\pi$ and $\eta\eta \rightarrow \pi\pi$ scattering, where the $f_0(500)$ is generated. Actually, the role of this resonance for $O(4)$ restoration has been recently emphasized in [23, 24]. Similarly, the r.h.s. of (6)–(7) and (9) connect with the $a_0(980)$ and $\kappa(800)$ resonances produced in $\pi\eta(\bar{K}K) \rightarrow \pi\eta$ and $\pi K(\pi\eta) \rightarrow \pi K$ scattering, respectively. The r.h.s of (10)–(14) include the effect of the η' , which couples through $A(x)$ to the $U(3)$ formulation of the chiral Lagrangian [10]. For instance (10) can be expressed in terms of $\pi\eta(\eta') \rightarrow \pi\eta(\eta')$ and $\bar{K}K \rightarrow \pi\eta(\eta')$ processes, all in the $a_0(980)$ channel.

In a simplified description based on the $O(4)$ model, $\sigma_l = N_\sigma \tilde{\sigma}$ and $\pi^a = N_\pi \tilde{\pi}^a$ with $(\tilde{\sigma}, \tilde{\pi}^a)$ the $O(4)$ vector field, $N_\sigma^2 = -M_\pi^2 \langle \bar{q}q \rangle_l / \hat{m}$ from the QCD mass term and $N_\pi^2 = -\langle \bar{q}q \rangle_l G_\pi^{-1}(0)$ from the WI $\chi_\pi = -\langle \bar{q}q \rangle_l / \hat{m}$ [21] with $G_\pi(p)$ the pion propagator. Those normalizations impose that the r.h.s of (5)–(11) should vanish at strict $O(4)$ restoration with vanishing $\langle \bar{q}q \rangle_l$.

Our results above lead to interesting consequences for the $I = 1/2$ sector. From the WI in (4), K - κ become degenerate at strict chiral restoration, since $\chi_S^\kappa(T) = \chi_P^K(T)$ for $\hat{m} \rightarrow 0$ and $\langle \bar{q}q \rangle_l \rightarrow 0$. This is consistent with the vanishing of the r.h.s of (9) in the $O(4)$ -model pion normalization and with K - κ bilinears being related by $SU(2)_A$ rotations. They can be rotated among them also by $U(1)_A$ rotations, which is expressed by the WI (14). In the physical crossover regime, (4) implies that the measure of the breaking of $I = 1/2$ degeneracy is precisely dictated by the subtracted quark condensate $\Delta_{l,s} = \langle \bar{q}q \rangle_l - 2(\hat{m}/m_s) \langle \bar{s}s \rangle$. This condensate is used customarily as order parameter in lattice analyses to avoid finite-size divergences [3] and its value at the chiral transition is typically reduced by one half with respect to the $T = 0$ estimate [2, 3]. The asymptotic K - κ degeneration observed for lattice screening masses [26] also confirms our conclusions.

Further remarkable results can be extracted from the crossed ls correlators, which are nonzero below the phase transition due to the $\eta - \eta'$ and $f_0(500) - f_0(980)$ mixing. In the former case, this is true even at the chiral limit [27].

In fact, one might also expect that they should vanish at $O(4)$ restoration also from chiral rotations: since the η_s and σ_s fields are invariant under $SU(2)_A$ transformations, $P_{ls} \rightarrow \langle \delta\eta_s \rangle$ under a $O(4)$ rotation, which should vanish by parity. In the same way $S_{ls} \rightarrow 0$ at the chiral transition. Thus, $P_{ls} \stackrel{O(4)}{\sim} 0$ and $S_{ls} \stackrel{O(4)}{\sim} 0$ are two additional chiral restoring conditions to be tested in the lattice. This result has an important consequence for partner degeneration since P_{ls} is connected through (3) with $\chi_{5,disc}$, leading to

$$\chi_{5,disc} \stackrel{O(4)}{\sim} 0. \quad (15)$$

Note that $\chi_{5,disc}$ measures $O(4) \times U(1)_A$ restoration, which hence becomes the pattern predicted by our WI analysis at exact chiral restoration, i.e. for two massless flavours and vanishing $\langle \bar{q}q \rangle_l$. This is consistent with the lattice results in [18, 19] and with the vanishing of (7)–(8) and (10)–(11) in the $O(4)$ model description.

In the physical massive case, it is then natural to expect the temperature scaling of $\chi_{5,disc}$ to be dictated by $\langle \bar{q}q \rangle_l$. To test this claim, we compare in Fig. 1a the T scaling of $\chi_{5,disc}$ and $\Delta_{l,s}$ using the lattice data in [3]. This plot provides a clear sign of correlation between the scaling of both quantities, especially near the critical region. We have also represented in Fig. 1a the behavior of $\sqrt{\Delta_{l,s}}$, also reasonably close to $\chi_{5,disc}$, as predicted by the $O(4)$ model pion normalization and (7). In this scenario, the $U(1)_A$ breaking at the crossover would be directly related to the condensate size. Alternatively, the anomalous contributions in the processes given by the r.h.s of (10)–(14) compensate the chiral suppression in the massive case.

In addition, (12) and (13) impose further conditions at $U(1)_A$ restoration, namely $P_{ls} \stackrel{U(1)_A}{\sim} S_{ls}$ and $P_{ss} \stackrel{U(1)_A}{\sim} S_{ss}$. The latter can be checked in the lattice using the data in [3] for the $\bar{s}s$ channel. The comparison is depicted in Fig. 1b and shows a clear sign of degeneracy around the asymptotic $U(1)_A$ restoration regime in [3], thus confirming our present analysis.

Our WI analysis also provides relevant conclusions regarding $\eta - \eta'$ mixing. In the simplest approach, the mixing angle $\theta_P(T)$ is defined to cancel the correlator

$$P_{\eta\eta'} = \frac{1}{6} (2P_{ss} - P_{ll} - 8P_{ls}) \sin 2\theta_P + \frac{\sqrt{2}}{3} (P_{ll} - 2P_{ss} - P_{ls}) \cos 2\theta_P = 0. \quad (16)$$

A relevant limit is ideal mixing, $\sin \theta_P^{id} = -\sqrt{2/3}$, so that $\eta \sim \eta_l$, $\eta' \sim \sqrt{2}\eta_s$. That limit is reached at $T = 0$ only when the anomalous contribution to the η' mass vanishes [27], formally achieved at $N_c \rightarrow \infty$ [10]. Thus it is natural to expect $\theta_P \rightarrow \theta_P^{id}$ in the temperature regime where $U(1)_A$ is restored. This is consistent with the experimentally observed reduction of the η' mass at finite

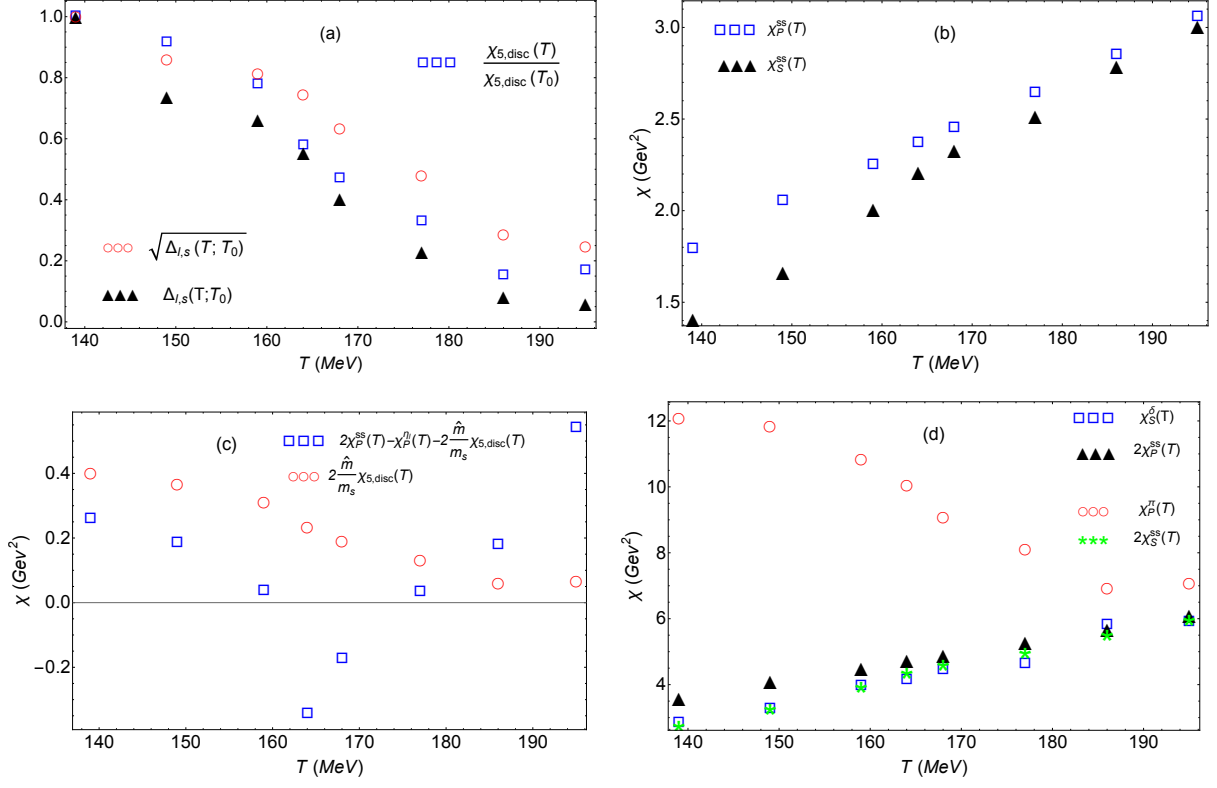


FIG. 1: Different susceptibilities combinations from the lattice data in [3] for $32^3 \times 8$ lattice size. (a): Comparison between the scaling of $\chi_{5, disc}$ and the subtracted quark condensate $\Delta_{l,s}(T; T_0) = \Delta_{l,s}(T)/\Delta_{l,s}(T_0)$ with respect to the reference temperature $T_0 = 139$ MeV. (b): Scalar and Pseudo-scalar pure strange susceptibilities. (c): Susceptibility combination related to the vanishing of the $\eta - \eta'$ mixing angle with $\hat{m}/m_s = 0.088$ [3], where we also plot $-\chi_P^{ls}$ according to (3). (d): Partner degeneration in the scenario where the two parameters in (c) remain small.

T [11] and with recent model analyses showing asymptotic ideal mixing at finite T [17, 28]. Ideal mixing is actually an additional consequence of our present model-independent analysis. From (16), $\theta_P = \theta_P^{id} \Leftrightarrow P_{ls} = 0$ but $P_{ll} - 2P_{ss} \neq 0$. Thus, (3) implies ideal mixing at $O(4) \times U(1)_A$ restoration. Note that $U(1)_A$ degenerates scalar and pseudoscalar partners, so the mixing angle in the scalar sector $\sigma_{l,s}$ degenerates with θ_P in that regime.

Another limit that deserves some comments is $\theta_P = 0$. From (16), $\theta_P = 0 \Leftrightarrow P_{ll} - 2P_{ss} = P_{ls} \neq 0$. Note that [21]:

$$\begin{aligned} \chi_P^{ll} - 2\chi_P^{ss} - \chi_P^{ls} = & -\frac{1}{\hat{m}} \langle \bar{q}q \rangle_l + \frac{2}{m_s} \langle \bar{s}s \rangle \\ & + 2 \frac{(\hat{m} - m_s)(\hat{m} + 2m_s)}{m_s^2} \chi_{5, disc}, \end{aligned} \quad (17)$$

where we have used (3) and the WIs in [21]. It is plausible that (17) reaches small values around chiral restoration, since $-\langle \bar{q}q \rangle_l(T)$ and $\chi_{5, disc}(T)$ decrease and $-\langle \bar{s}s \rangle(T)$ smoothly increases. Actually, we plot this combination in Fig. 1c using again the lattice data in [3]. The neat separation between $O(4)$ and $O(4) \times U(1)_A$ found in that work guarantees $\chi_P^{ls} \neq 0$ around chiral restoration and hence a vanishing mixing angle regime. We see in Fig. 1c

that there is actually a $\theta_P(T) \sim 0$ region close to chiral restoration, where the combination (3) develops a minimum. For higher T , θ_P moves from zero to θ_P^{id} asymptotically, as we have just commented. Note also that (17) vanishes in the $SU(3)$ limit, i.e. $m_s \rightarrow \hat{m}$ and $\langle \bar{s}s \rangle \rightarrow \langle \bar{q}q \rangle_l/2$, consistently with $\theta_P \rightarrow 0$ for $m_K = m_\pi$ at $T = 0$ [27].

Moreover, in the intermediate region between $O(4)$ and $O(4) \times U(1)_A$ restoration, if the combination in (17) and χ_P^{ls} happens to remain small, there is an additional sign of partner degenerations, namely $2P_{ss} \sim S_{\delta\delta}$ and $2S_{ss} \sim P_{\pi\pi}$. These two identities are tested for the same lattice data in Fig. 1d. However, if the susceptibility combination in Fig. 1c would keep on growing for higher T , the degeneration in Fig. 1d would not be maintained.

CONCLUSIONS

The analysis based on WIs performed here implies a $O(4) \times U(1)_A$ pattern in the limit of exact restoration (vanishing light quark mass and quark condensate), as well as ideal $\eta - \eta'$ mixing at the $O(4) \times U(1)_A$ transition.

On the massive case, a vanishing pseudoscalar mixing is expected if there is a sizable transient regime between both $O(4)$ and $O(4) \times U(1)_A$ restoration. All these conclusions have been achieved by identifying the relevant correlator combinations. In addition, we have provided several useful results, testable in lattice simulations and in model analyses, connecting partner degeneration with specific meson vertices and processes. Available lattice data confirm our claims about chiral patterns, mixing angles, and new partner degeneracies concerning strange and light-strange correlators. In particular, $K - \kappa$ degeneration is directly linked to the subtracted lattice quark condensate.

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